

PUBLICATION No 124



AD615481

SHIPBUILDING LABORATORY

TECHNOLOGICAL UNIVERSITY - DELFT

COPY	<u>2</u>	OF	<u>3</u>	by
HARD COPY	\$. 2 . 0 0			
MICROFICHE	\$. 0 . 5 0			

THE DISTRIBUTION OF THE HYDRODYNAMIC FORCES *268*
ON A HEAVING AND PITCHING SHIPMODEL,
WITH ZERO FORWARD SPEED IN STILL WATER

BY

PROF. IR J. GERRITSMA AND W. BEUKELMAN

SPONSORED BY THE OFFICE OF NAVAL RESEARCH
CONTRACT No. N 62558-4097



FEBRUARY 1965

ARCHIVE COPY

**CLEARINGHOUSE FOR FEDERAL SCIENTIFIC AND TECHNICAL INFORMATION, CFSTI
INPUT SECTION 410.11**

**LIMITATIONS IN REPRODUCTION QUALITY OF TECHNICAL ABSTRACT BULLETIN
DOCUMENTS, DEFENSE DOCUMENTATION CENTER (DDC)**

AD 615481

- ☐ 1. AVAILABLE ONLY FOR REFERENCE USE AT DDC FIELD SERVICES.
COPY IS NOT AVAILABLE FOR PUBLIC SALE.
- ☒ 2. AVAILABLE COPY WILL NOT PERMIT FULLY LEGIBLE REPRODUCTION.
REPRODUCTION WILL BE MADE IF REQUESTED BY USERS OF DDC.
- ☒ A. COPY IS AVAILABLE FOR PUBLIC SALE.
- ☐ B. COPY IS NOT AVAILABLE FOR PUBLIC SALE.
- ☐ 3. LIMITED NUMBER OF COPIES CONTAINING COLOR OTHER THAN BLACK
AND WHITE ARE AVAILABLE UNTIL STOCK IS EXHAUSTED. REPRODUCTIONS
WILL BE MADE IN BLACK AND WHITE ONLY.

TSL-121-2/65

DATE PROCESSED:

PROCESSOR:



SHIPBUILDING LABORATORY

TECHNOLOGICAL UNIVERSITY – DELFT

THE DISTRIBUTION OF THE HYDRODYNAMIC FORCES
ON A HEAVING AND PITCHING SHIPMODEL,
WITH ZERO FORWARD SPEED IN STILL WATER

BY

PROF. IR J. GERRITSMAN AND W. BEUKELMAN

SPONSORED BY THE OFFICE OF NAVAL RESEARCH
CONTRACT No. N 62558-4097

THE DISTRIBUTION OF THE HYDRODYNAMIC FORCES ON A
HEAVING AND PITCHING SHIPMODEL, WITH ZERO FORWARD SPEED IN STILL WATER.

by Prof. ir J. Gerritsma and W. Beukelman.

Summary.

As an extension of earlier work on the distribution of the hydrodynamic forces, acting on a heaving and pitching shipmodel, this report deals with the case of zero forward speed.

Forced oscillation tests were carried out with a segmented shipmodel to investigate the distribution of the forces along the hull for heaving and pitching motions. The in-phase and quadrature components of the vertical forces on each of the seven sections were measured as a function of frequency and an analysis was made of their distribution along the length of the shipmodel.

The experimental results are compared with Grim's values for damping and added mass. The comparison shows a satisfactory agreement between theory and experiment.

1. Introduction.

According to strip-theory the damping coefficients, the added mass and the added mass-moment of inertia of a ship, performing harmonic heaving and pitching motions in still water, can be calculated by integrating the corresponding two-dimensional quantities for each cross-section over the length of the ship.

This very simple procedure does not include three-dimensional or forward speed effects on the total damping and added mass. Forced oscillation tests with a 2.3 meter model of a normal cargoship have shown a very small speed dependency of these quantities, at least for frequencies of interest for the usual seakeeping problems [1]. The speed range was $F_n = .15$ to $F_n = .30$. The tests were carried out in the 4.3 meter wide towing tank of the Delft Shipbuilding Laboratory. Tests at zero forward speed were not considered in this tank because a large wall influence was expected from the reflection of waves, generated by the forced model motions. Without zero speed experiments form effects and speed effects could not be separated.

In a later stage it was decided to carry out zero speed tests in a wide basin to minimize wall interference. The results could then be used to study form effect in damping and added mass by comparing the results with calculations from strip theory. (The zero speed experiments could also double the range of speeds considered).

This is of interest for the analysis of the damping cross-coupling coefficients of the motion equations. These cross-coupling coefficients depend quite strongly on the speed of the ship and this is important for the calculation of ship motions in waves. From strip-theory [1] it follows that:

$$\left. \begin{aligned} e &= \int_L N' x dx - V_m \\ E &= \int_L N' x dx + V_m \end{aligned} \right\} \quad (1)$$

where:

- e is the coefficient of the pitch velocity term in the motion equation for heave,
- E is the coefficient of the heave velocity term in the equation for pitch,
- N' is the sectional damping coefficient,
- m is the total added mass,
- V is the forward speed of the ship.

By plotting e and E on a base of forward speed two straight lines correspond to each frequency of motion. The two lines have a common point for $V=0$, where: $e = E = \int_L N' x dx$ and their slope is given by $\pm m$.

The experiments have shown the validity of equation (1) in the speed range $Fn = 0.15 - .30$. The zero forward speed tests could give additional support to the theory because of the considerable extension of the speed range.

Funds became available from the Office of Naval Research by which forced oscillation tests with a segmented model at zero speed could be carried out in the Seakeeping Laboratory of the Netherlands Ship Model Basin at Wageningen.

In this report the results of the experiments are given. The results are compared with calculated values, using Grim's two-dimensional results for damping and added mass for ship like sections [2].

2. Experiments.

The oscillation tests were carried out with a 2.3 meter model of the Sixty Series, having a block coefficient $C_B = 0.70$. The main dimensions are given in Table I. The model is made of polyester, re-inforced with fibreglass, and consists of seven separate sections of equal length. Each of the sections has two end-bulkheads. The width of the gap between two sections is one millimeter. The sections are not connected to each other, but they are kept in position by means of stiff strain-gauge dynamometers, which are connected to a longitudinal steel box girder above the model.

The dynamometers are sensitive only for forces perpendicular to the baseline of the model.

By means of a Scotch-Yoke mechanism a harmonic heaving or pitching motion can be given to the combination of the seven sections, which form the shipmodel. The total forces on each section could be measured as a function of frequency.

A non segmented model of the same form was also tested in the same conditions of frequency to compare the forces on the whole model with the sum of the section results. A possible effect of the gaps between the sections could be detected in this way. The arrangement of the tests with the segmented model and with the whole model is given in Fig. 1.

The mechanical oscillator and the measuring system is shown in Fig. 2. In principle the measuring system is similar to the one described by Goodman [3]: the measured force signal is multiplied by $\cos \omega t$ and $\sin \omega t$ and after integration the first harmonics of the in-phase and quadrature components can be found without distortion due to vibration noise. In some details the electronic circuit differs somewhat from the description in [3]. In particular synchro resolvers are used instead of sine-cosine potentiometer, because they allow higher rotational speeds.

The accuracy of the instrumentation proved to be satisfactory which is important for the determination of the quadrature components, which are small in comparison with the in-phase components of the measured forces. Throughout the experiments only first harmonics were determined. It should be noted that non-linear effects may be important for the sections at the bow and the stern where the ship is not wall-sided. The forced oscillation tests were carried out for frequencies up to $\omega = 14$ rad/sec.

The motion amplitudes of the shipmodel covered a sufficiently large range to study the linearity of the measured values. An example of the measured forces on section 2, when the combination of the seven sections performs a pitching motion, is given in Fig. 3.

The position of the model in the tank is given in Figure 4. There are beaches on two adjacent sides of the tank and a wire mesh along a third side as indicated in the figure.

Each oscillation test was completed within the time that observable reflected waves reached the model.

TABLE 1. Main particulars of the shipmodel.

Length between perpendiculars	2.258 m
Length on the waterline	2.296 m
Breadth	0.322 m
Draught	0.129 m
Volume of displacement	0.0657 m ³
Blockcoefficient	0.700
Coefficient of mid length section	0.986
Prismatic coefficient	0.710
Waterplane area	0.572 m ²
Waterplane coefficient	0.785
Longitudinal moment of inertia of waterplane	0.1685 m ³
L.C.B. forward of $L_{pp}/2$	0.011 m
Centre of effort of waterplane after $L_{pp}/2$	0.038 m
Froude number of service speed	0.20

3. Presentation of the results.

3.1. Whole model.

It is assumed that the force F and the moment M acting on a forced heaving and pitching shipmodel in still water can be described by the following equations:

Heave:

$$\left. \begin{aligned} (a + \rho \nabla) \ddot{z}_0 + b \dot{z}_0 + c z_0 &= F_z \sin(\omega t + \alpha) \\ D \ddot{z}_0 + E \dot{z}_0 + G z_0 &= -M_z \sin(\omega t + \beta) \end{aligned} \right\} \quad (2)$$

Pitch:

$$\left. \begin{aligned} (A + k_{yy}^2 \rho \nabla) \ddot{\theta} + B \dot{\theta} + C \theta &= M_\theta \sin(\omega t + \gamma) \\ d \ddot{\theta} + e \dot{\theta} + g \theta &= -F_\theta \sin(\omega t + \delta) \end{aligned} \right\} \quad (3)$$

where: z_0 is the heave displacement and θ is the pitch angle.

For a given heaving motion: $z_o = z_a \sin \omega t$, it follows that:

$$\left. \begin{aligned} b &= \frac{F_z \sin \alpha}{z_a \omega} \\ a &= \frac{c z_a - F_z \cos \alpha}{z_a \omega^2} - \rho \nabla \\ E &= \frac{-M_z \sin \beta}{z_a \omega} \\ D &= \frac{g z_a + M_z \cos \beta}{z_a \omega^2} \end{aligned} \right\} \quad (4)$$

Similar expressions are valid for the pitching motion. The determination of the damping coefficients b and B and the damping cross-coupling coefficients e and E , is straight forward: for a given frequency these coefficients are proportional to the quadrature components of the forces or moments for unit amplitude of motion. For the determination of the added mass, the added mass moment of inertia, a and A , and the added mass cross-coupling coefficients d and D it is necessary to know the restoring force and moment coefficients c and C , and the statical cross-coupling coefficients g and G .

For zero speed of advance, these coefficients can be calculated easily from the known values of the waterplane area, the longitudinal moment of inertia of the waterplane area and the position of the centre of effort of the waterplane area, as given in Table 1.

The results for the whole model are given in Figures 5 and 6. These figures also give the sum of the results from each section to show that the combined effect of the slits between the sections and possible inaccuracies in the measurements is very small.

3.2. Results for the sections.

The components of the forces on each of the seven sections were determined in the same way as for the whole model. As only the forces and moments on the sections were measured two equations remain for each section:

Heave:

$$(a^* + \rho \nabla^*) \ddot{z}_0 + b^* \dot{z}_0 + c^* z_0 = F_z^* \sin(\omega t + \alpha^*)$$

Pitch:

$$(d^* + \rho \nabla^* x_i) \ddot{\theta} + j \dot{\theta} + g \theta = -F_\theta^* \sin(\omega t + \delta^*)$$

(5)

where $\rho \nabla^* x_i$ is the mass moment of the section i with respect to the pitching axis. The star (*) indicates the coefficients of the sections. The section coefficients divided by the length of the sections give the mean cross-section coefficients, thus:

$$\frac{a^*}{1/7 L_{pp}} = \bar{a}^*,$$

and so on. Assuming that the distributions of the cross-sectional values of the coefficients: a^* , b^* etcetera, are continuous curves, these distributions can be determined from the seven mean cross-section values.

In Figure 7 the distributions of the added mass a , the damping coefficient b and the cross-coupling coefficients d and e are given as a function of frequency.

Numerical values of the section results, a^* , b^* etcetera, are summarized in Table 2. In this Table the results for the whole model are also given.

The sum of the section results, as used in Figures 5 and 6 for comparison with the whole model results, are determined according to the following equations:

$$\left. \begin{array}{l} \sum a^* = a \\ \sum b^* = b \\ \sum d^* = d \\ \sum e^* = e \end{array} \right\} \begin{array}{l} \int_L^L d' x dx = A \\ \int_L^L e' x dx = B \\ \int_L^L a' x dx = D \\ \int_L^L b' x dx = E \end{array} \quad (6)$$

The values for A , B , D and E are given in Table 3.

TABLE 2. Added mass, damping coefficient and cross-coupling coefficients for the sections and for the whole model.

$F_D = 0$									
ω rad/ sec.	1	2	3	4	5	6	7	sum of sections	whole model
$a^* \quad \text{kg sec}^2/\text{m}$								a	
2	1.31	2.58	4.09	3.65	2.90	2.78	0.68	17.99	18.80
4	0.48	1.38	1.78	1.93	1.75	1.17	0.21	8.70	8.61
6	0.23	1.02	1.18	1.29	1.26	0.88	0.18	6.04	5.84
8	0.20	0.81	1.12	1.20	1.22	0.85	0.11	5.51	5.43
10	0.20	0.74	1.28	1.47	1.36	0.85	0.12	6.02	5.86
12	0.21	0.78	1.35	1.57	1.63	0.90	0.14	6.58	6.48
14	0.19	0.85	1.48	1.68	1.73	1.04	0.17	7.14	6.82
$b^* \quad \text{kg sec/m}$								b	
2	2.00	1.40	3.95	3.90	3.20	2.30	0.90	17.65	10.25
4	2.08	4.48	5.18	5.93	5.75	5.25	1.05	29.72	29.17
6	1.98	4.23	5.10	5.15	4.62	3.82	0.90	25.80	25.03
8	2.15	3.80	2.93	2.13	2.81	2.53	0.74	17.09	18.00
10	2.18	3.06	1.72	1.28	1.31	1.78	0.56	11.89	12.31
12	1.98	2.22	1.32	0.97	0.64	1.41	0.42	8.96	9.43
14	1.54	1.78	1.51	1.03	0.69	1.48	0.30	8.33	7.87
$d^* \quad \text{kg sec}^2$								d	
2	-0.55	-1.84	-1.38	-0.06	-1.12	-1.33	-0.31	-1.07	-1.15
4	-0.43	-0.99	-0.69	-0.03	0.68	0.80	0.24	-0.42	-0.48
6	-0.31	-0.68	-0.41	0.01	0.44	0.56	0.16	-0.23	-0.17
8	-0.25	-0.50	-0.33	0.00	0.36	0.45	0.13	-0.14	-0.09
10	-0.20	-0.43	-0.38	0.00	0.39	0.45	0.12	-0.05	-0.02
12	-0.17	-0.45	-0.41	0.00	0.41	0.51	0.12	+0.01	+0.04
14	-0.19	-0.49	-0.42	0.00	-0.42	-0.53	-0.15	0	+0.03
$e^* \quad \text{kg sec}$								e	
2	-	-0.07	-0.18	-	-	-	-	-	+0.13
4	-1.21	-1.28	-0.53	0.02	0.33	1.19	0.52	-0.46	-0.69
6	-1.99	-2.80	-1.82	0.00	1.61	2.10	0.81	-2.09	-1.81
8	-1.98	-2.60	-1.09	-0.01	0.81	1.62	0.72	-2.53	-2.46
10	-1.84	-2.96	-0.66	-0.01	0.41	0.95	0.54	-2.57	-2.56
12	-1.67	-1.45	-0.52	-0.01	0.30	0.69	0.45	-2.21	-2.08
14	-1.55	-1.19	-0.60	+0.09	0.30	-0.63	0.38	-1.94	-1.95

TABLE 3. Added mass moment of inertia, damping coefficient and cross-coupling coefficients for the whole model.

ω rad/ sec.	A kg sec ²		B kg m sec		D kg sec ²		E kg sec	
	sum of sections	whole model	sum of sections	whole model	sum of sections	whole model	sum of sections	whole model
2	-	3.77	-	0.44	-	-1.28	-	+0.55
4	2.26	2.34	3.72	3.72	-0.58	-0.41	-0.52	-0.63
6	1.54	1.49	6.94	6.90	-0.19	-0.18	-1.63	-1.83
8	1.18	1.13	5.83	5.65	-0.08	-0.06	-2.42	-2.49
10	1.13	1.09	4.50	4.47	+0.06	+0.02	-2.58	-2.64
12	1.18	1.17	3.53	3.29	+0.08	+0.03	-2.46	-2.32
14	-	1.22	-	2.84	-	+0.04	-	-2.07

For A and B the sum of the section results were obtained by using equation (6).

4. Analysis of the results.

The experimental values of the coefficients of the motion equations will now be analysed by using the strip-theory. For a description of the strip-theory reference is made to [1]. In the special case of zero forward speed the coefficients of the motion equations, according to the strip-theory, have a very simple form, as shown in Table 4, where:

a . . . g } Coefficients of the motion equations
A . . . G } (hydromechanical part).

m' Added mass of a cross-section (zero speed)

N' Damping coefficient of a cross-section (zero speed).

A_w Waterplane area.

S_w Statical moment of waterplane.

I_w Longitudinal moment of inertia of waterplane.

TABLE 4. Coefficients of the motion equations according to strip-theory for zero forward speed.

$F_n = 0.$	
$a = \int_L m' dx$	$d = \int_L m' x dx$
$b = \int_L N' dx$	$e = \int_L N' x dx$
$c = \int g A_w$	$g = \int g S_w$
$A = \int_L m' x^2 dx$	$D = \int_L m' x dx$
$B = \int_L N' x^2 dx$	$E = \int_L N' x dx$
$C = \int g I_w$	$G = \int g S_w$

Table 4 shows that each of the coefficients can be calculated by integrating the corresponding cross-sectional values over the length of the ship.

This procedure has been carried out for the model under consideration. Grim's cross-sectional values for the damping coefficients N' and the added mass m' were used.

In Figure 7 the calculated distributions of the added mass and the damping coefficients are shown in comparison with the experimental values. In general there is satisfactory agreement between theory and experiment. The calculated added mass at the lowest frequencies is smaller than indicated by the experiments, and some of the distributions show a slight shift in longitudinal direction when compared with the measured values.

Also the values of the total added mass, the damping coefficients and the cross-coupling coefficients are also compared in the Figures 8 and 9.

The numerical values are summarized in Table 5.

TABLE 5. Comparison of calculated and measured coefficients.

	A kg m sec ²		B kg m sec		a kg sec ² /m		b kg sec/m		D	d	D/d	E	e	E/e
	ω rad/ sec.	measured	calculated	measured	calculated	measured	calculated	measured	calculated	measured	calculated	measured	calculated	measured
2	3.77	3.03	0.44	6.12	18.80	12.45	10.25	25.37	-1.28	-1.15	-0.64	+0.55	+0.13	-1.38
4	2.34	1.60	3.72	8.29	8.61	6.73	29.17	31.66	-0.41	-0.48	-0.33	-0.63	-0.69	-2.12
6	1.49	1.15	6.90	7.64	5.84	5.25	25.03	26.22	-0.18	-0.17	-0.19	-1.83	-1.81	-2.54
8	1.13	1.09	5.65	5.81	5.43	5.41	18.00	16.91	-0.06	-0.09	-0.10	-2.49	-2.46	-2.66
10	1.09	1.18	4.47	4.01	5.86	6.04	12.31	9.46	+0.02	-0.02	-0.05	-2.64	-2.56	-2.47
12	1.17	1.28	3.29	2.69	6.43	6.61	9.43	5.14	+0.03	+0.04	-0.04	-2.32	-2.08	-2.08
14	1.22	1.37	2.84	1.84	6.82	7.00	7.87	2.95	+0.04	+0.03	-0.04	-2.07	-1.95	-1.65

In the region of resonance (for heave $\omega_n = 6.9$ rad/sec, for pitch $\omega_n = 7.0$ rad/sec) the agreement between theory and experiment is excellent.

For frequencies lower than 5 rad/sec the calculated pitch damping coefficient and the added mass moment of inertia show a rather large difference between calculation and experiment. The heave and pitch damping coefficients for $\omega > 10$ rad/sec are smaller than the measured values.

Although only first harmonic components of the hydromechanical forces were measured, there is no guarantee that the 90 degrees out of phase components correspond exclusively to wave damping. Particularly at high frequencies damping of viscous origin could be present and the first harmonic content of such a non wave making damping could influence the final result. Because the calculation only considers pure wave damping this could be a reason for the observed differences.

For practical purposes, such as the calculation of ship motions in waves, the differences as shown in the Figures 8 and 9 are not of much interest.

As a final comparison between experiment and strip-theory, the calculated and measured damping cross-coupling coefficients e and E at zero forward speed, are plotted in Figure 10, which is taken from reference [1]. In Figure 10 it is shown that e and E vary linearly with forward speed at constant frequency of motion. The range of forward speed where this is valid now includes zero speed and the calculated damping cross-coupling coefficients are in agreement with the measurements.

5. Acknowledgement.

The work described in this report was sponsored by the Office of Naval Research under Contract No. N 62558 - 4097.

The Authors are indebted to Mr. T.R. Dyer for his assistance in carrying out the experiments.

6. References.

1. J. Gerritsma and W. Beukelman.

"The distribution of the hydrodynamic forces on a heaving and pitching ship model in still water".

Paper presented at the Fifth Symposium of Naval Hydrodynamics, Bergen, Norway - 1964.

International Shipbuilding Progress 1964.

2. O. Grim.

"A method for a more precise computation of heaving and pitching motions both in smooth water and in waves".

Third Symposium of Naval Hydrodynamics, Scheveningen - 1960.

3. A. Goodman.

"Experimental techniques and methods of analysis used in submerged body research".

Third Symposium of Naval Hydrodynamics, Scheveningen - 1960.

7. List of symbols.

$a \dots g$ } Coefficients of the motion equations.
 $A \dots G$ } (hydromechanical part).

$a^* \dots g^*$ } The same for a section of the ship.
 $A^* \dots G^*$ }

$a' \dots g'$ } The same for a cross-section of the ship.
 $A' \dots G'$ }

A_w Waterplane area.

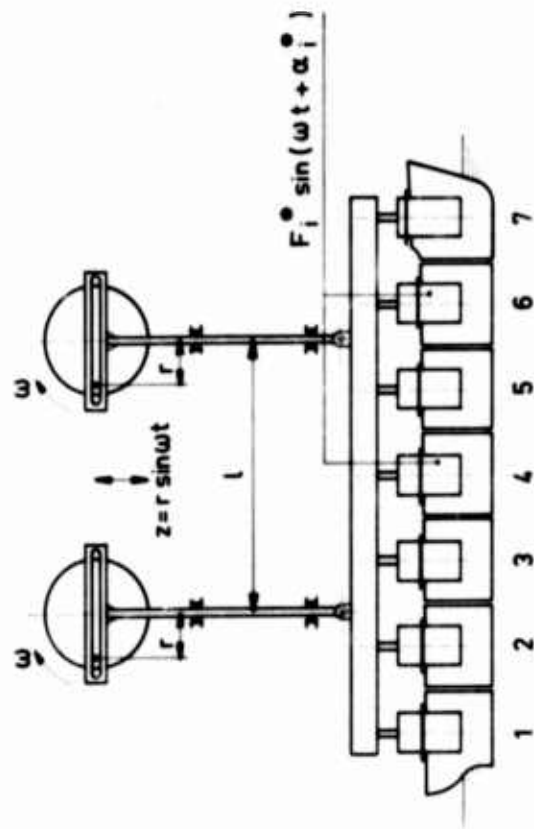
C_B Blockcoefficient.

F_n Froude number.

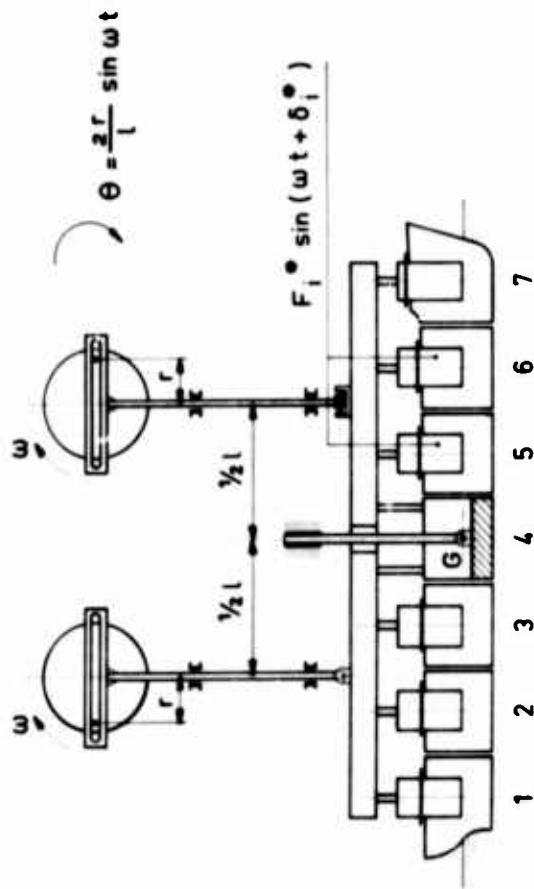
F_z, F_0 Amplitude of vertical force on a heaving or pitching ship.

g Acceleration of gravity.

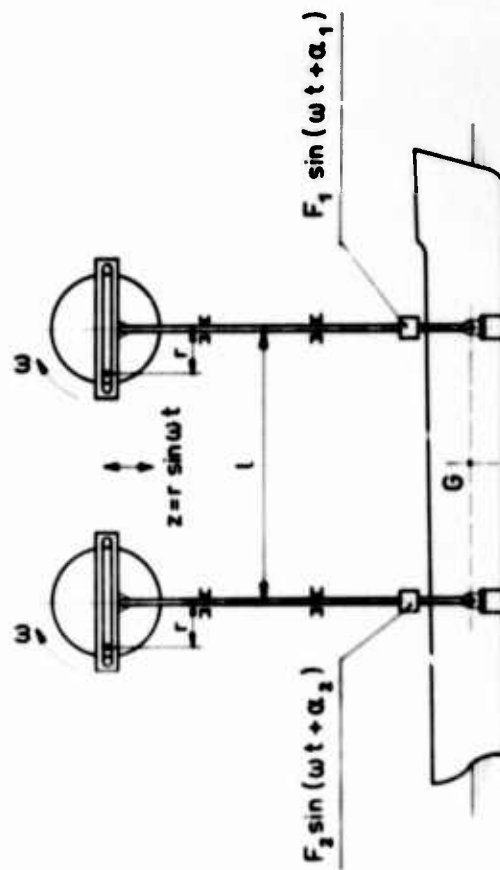
I_w	Longitudinal moment of inertia of waterplane
k_{yy}	Longitudinal radius of inertia of the ship.
L_{pp}	Length between perpendiculars.
M_z, M_0	Amplitude of moment on a heaving or pitching ship.
m'	Added mass of a cross-section (zero speed).
m	Added mass of a ship (heave).
N'	Damping coefficient of a cross-section (zero speed).
S_w	Statical moment of waterplane.
t	Time.
V	Forward speed of ship.
x, y, z	Right hand coordinate system, fixed to the ship.
x_0, y_0, z_0	Right hand coordinate system, fixed in space.
z_0	Vertical displacement of ship.
z_a	Heave amplitude.
x_1	Horizontal distance of centre of gravity of a section to the pitching axis.
$\alpha, \beta, \gamma, \delta$	Phase angles.
θ	Pitch angle.
θ_a	Pitch amplitude.
ρ	Density of water.
ω	Circular frequency.
∇	Volume of displacement of ship.
∇'	Volume of displacement of section.



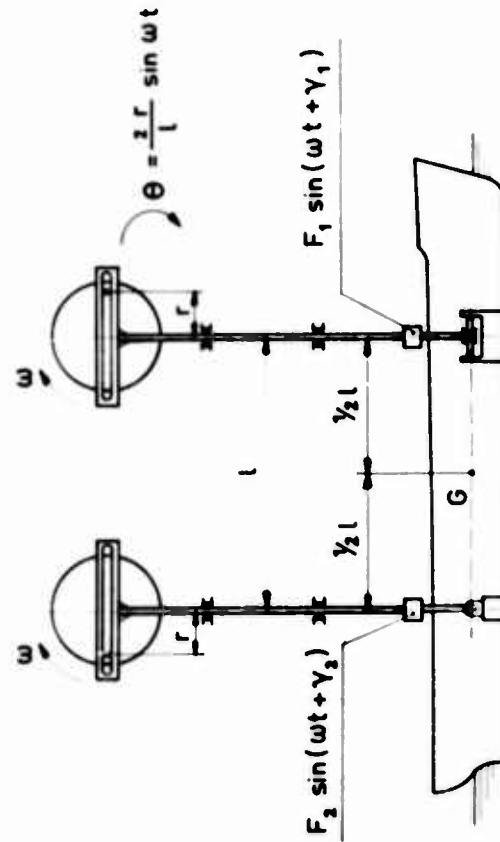
HEAVING TEST WITH SEGMENTED MODEL



PITCHING TEST WITH SEGMENTED MODEL



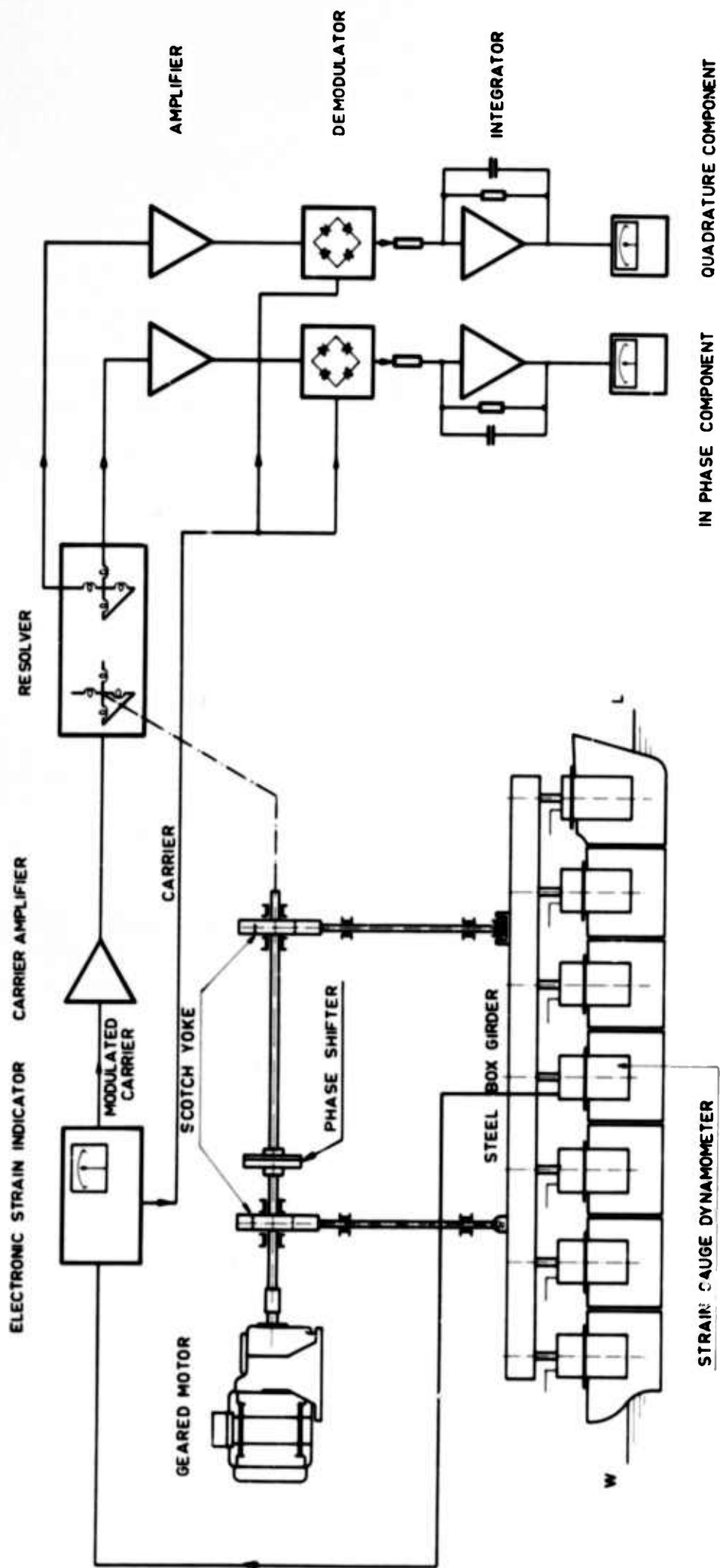
HEAVING TEST WITH WHOLE MODEL



PITCHING TEST WITH WHOLE MODEL

ARRANGEMENT OF OSCILLATION TESTS

FIGURE 1



PRINCIPLE OF MECHANICAL OSCILLATOR AND ELECTRONIC CIRCUIT

FIGURE 2

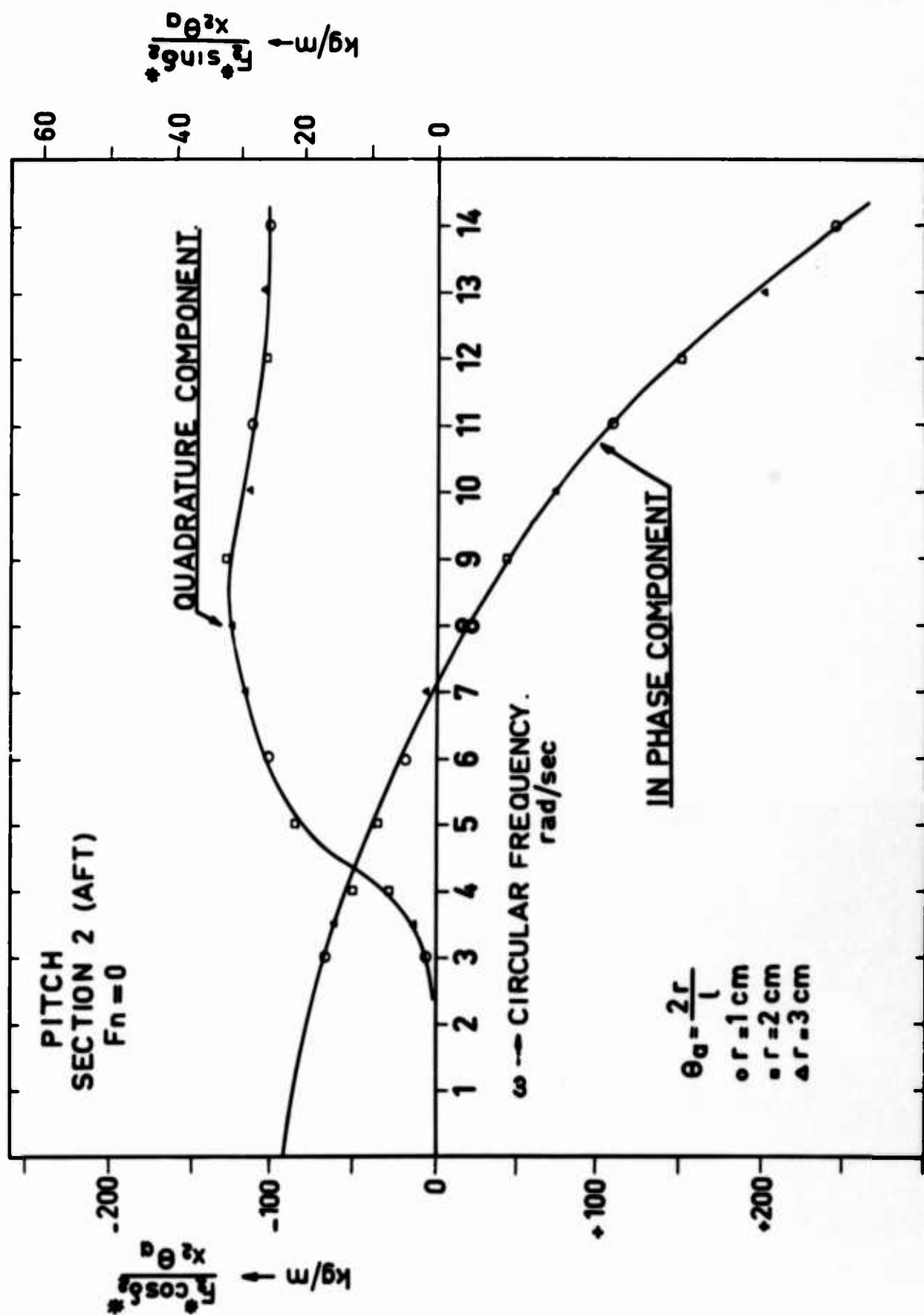


FIG.3 COMPONENTS OF FORCE ON SECTION 2. PITCHING MOTION

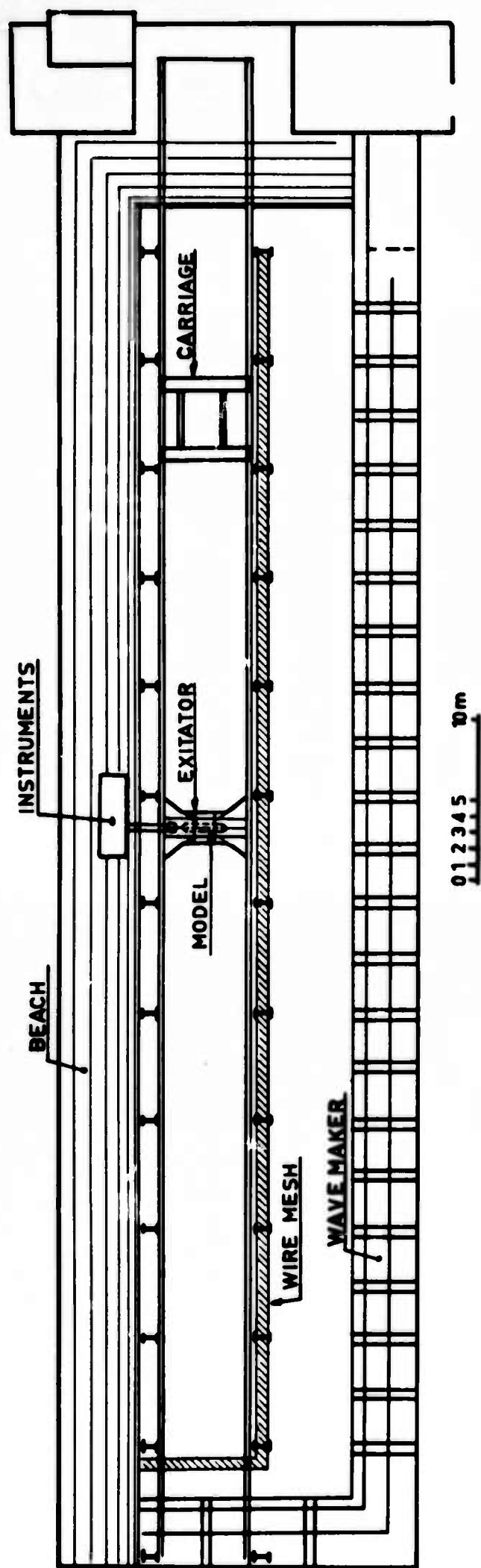


FIG. 4 POSITION OF MODEL IN SEAKEEPING BASIN

HEAVING MOTION

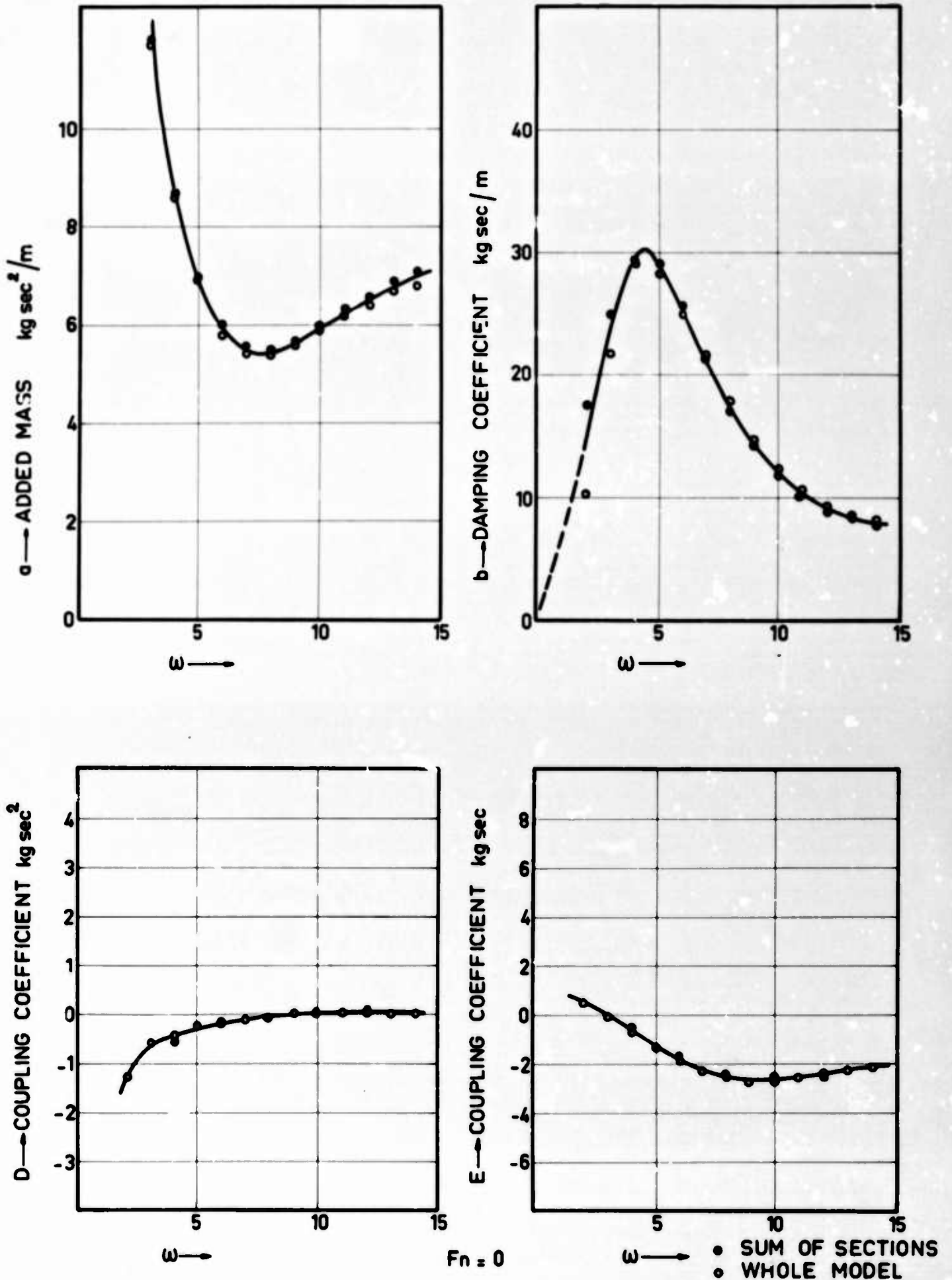


FIG. 5 EXPERIMENTAL RESULTS FOR 'WHOLE MODEL'

PITCHING MOTION

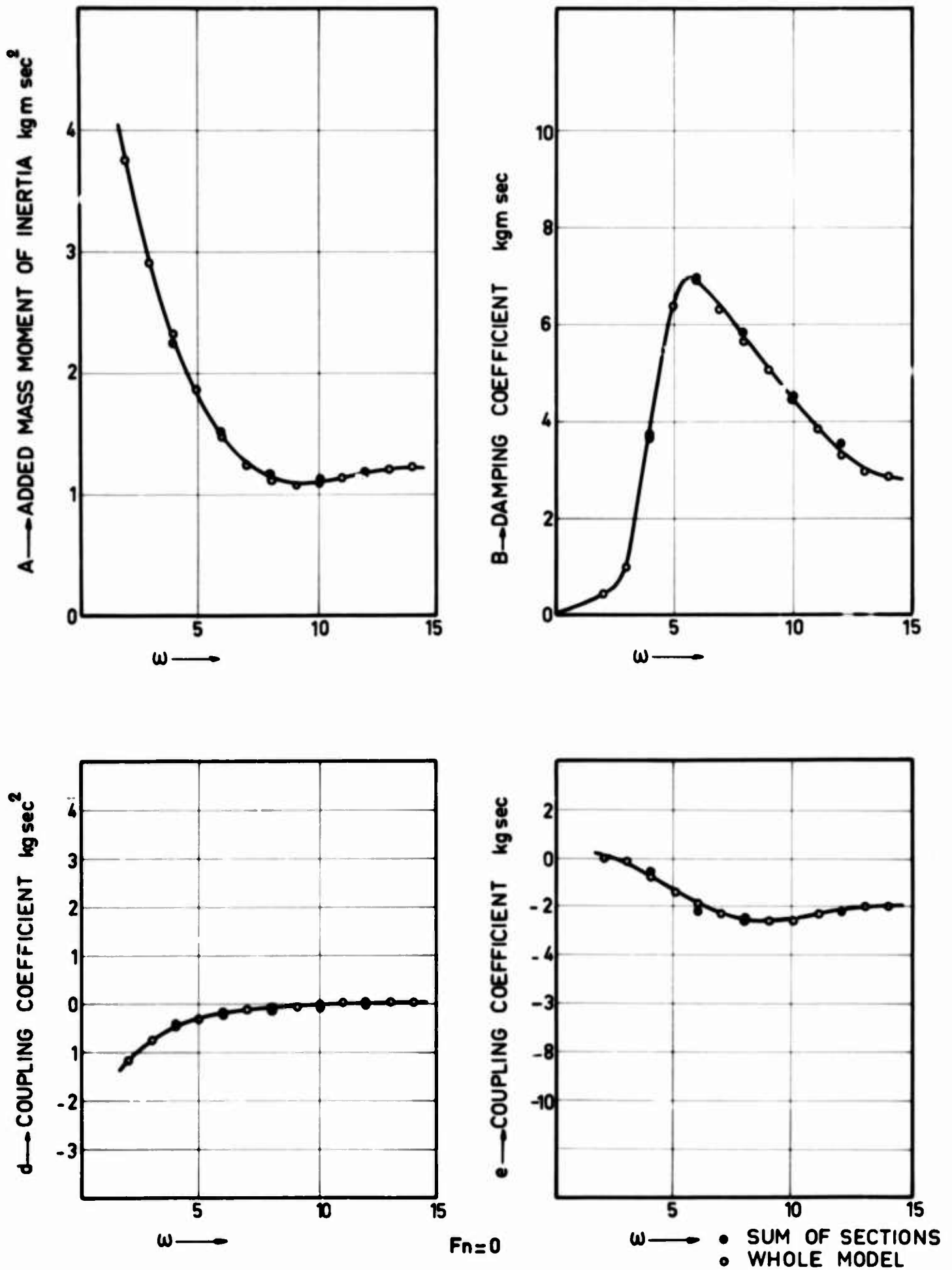


FIG. 6 EXPERIMENTAL RESULTS FOR WHOLE MODEL

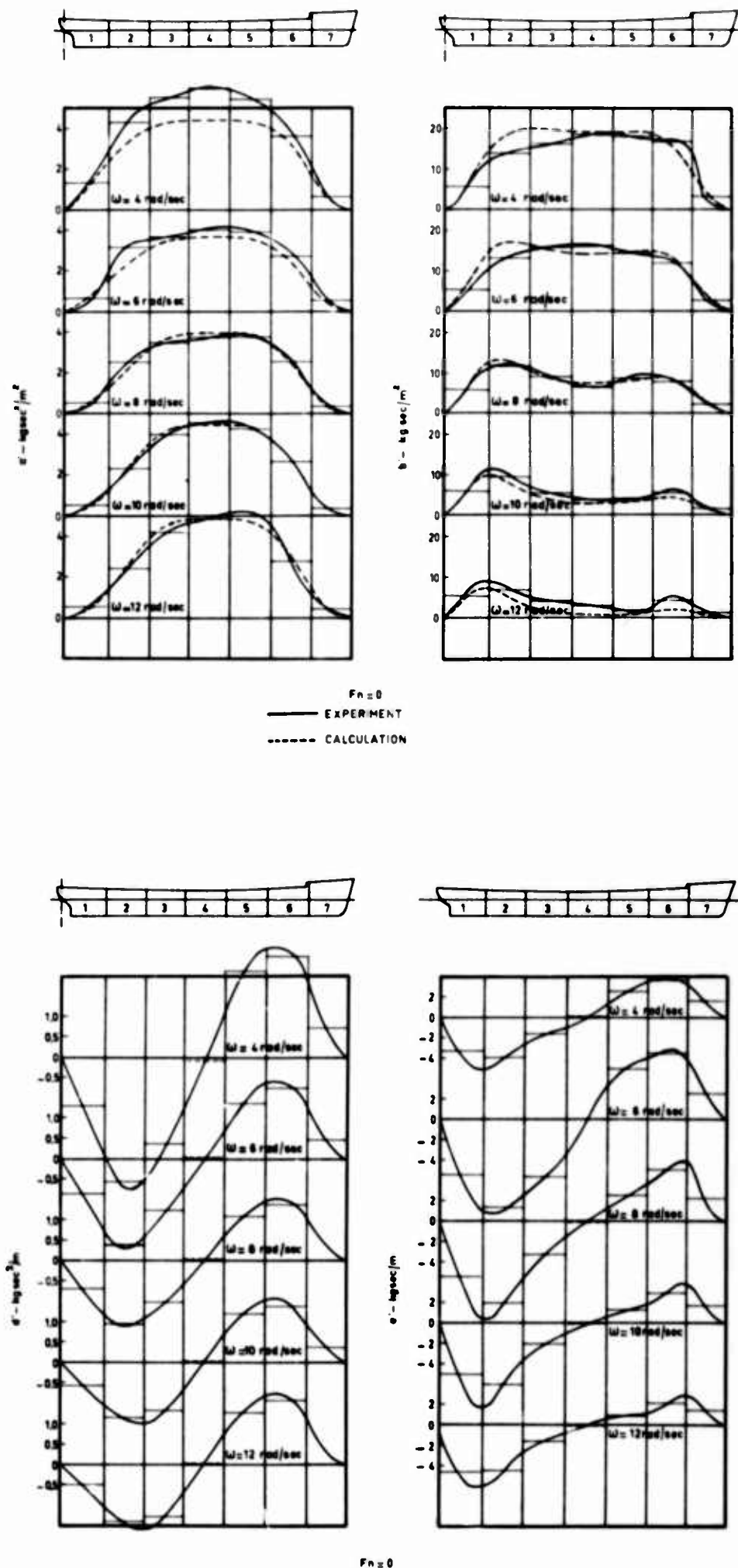
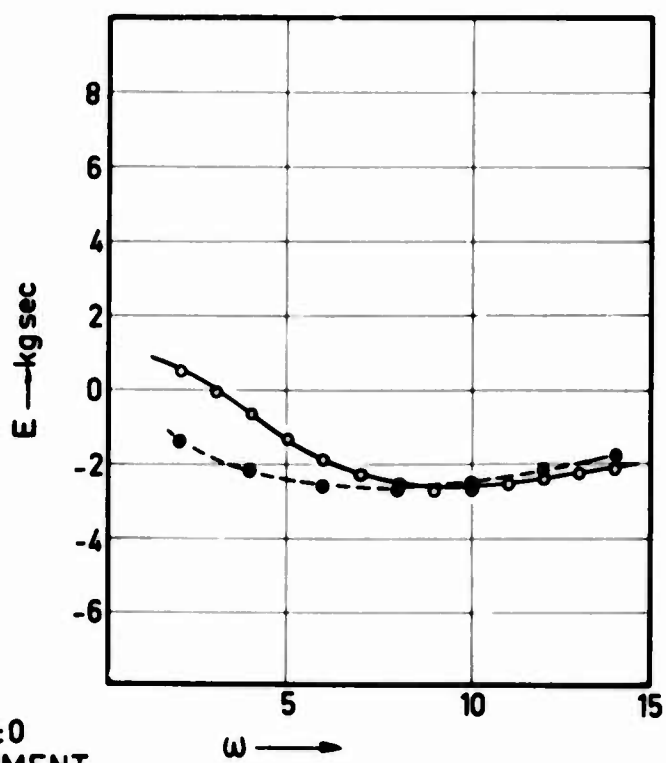
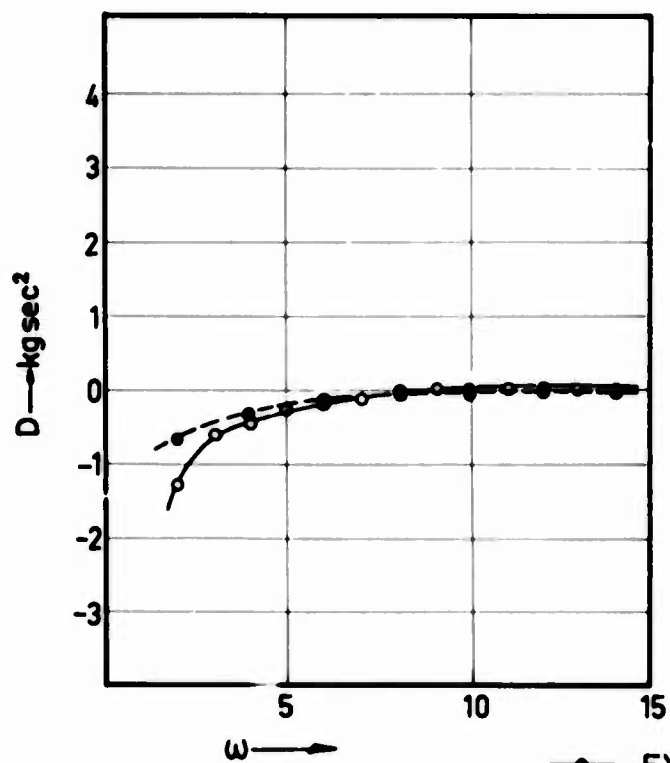
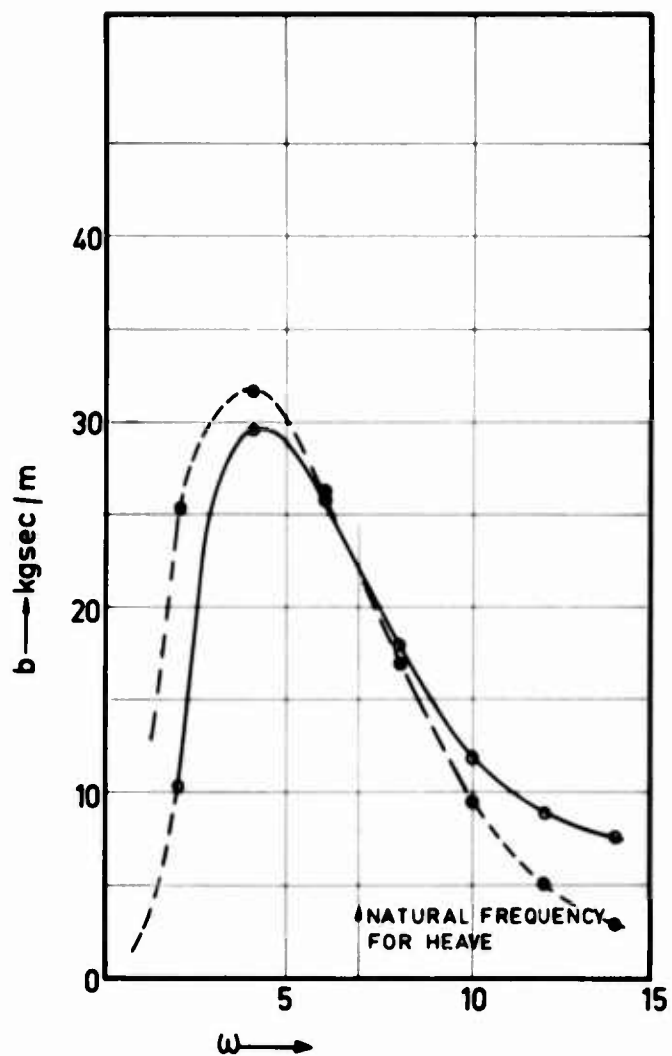
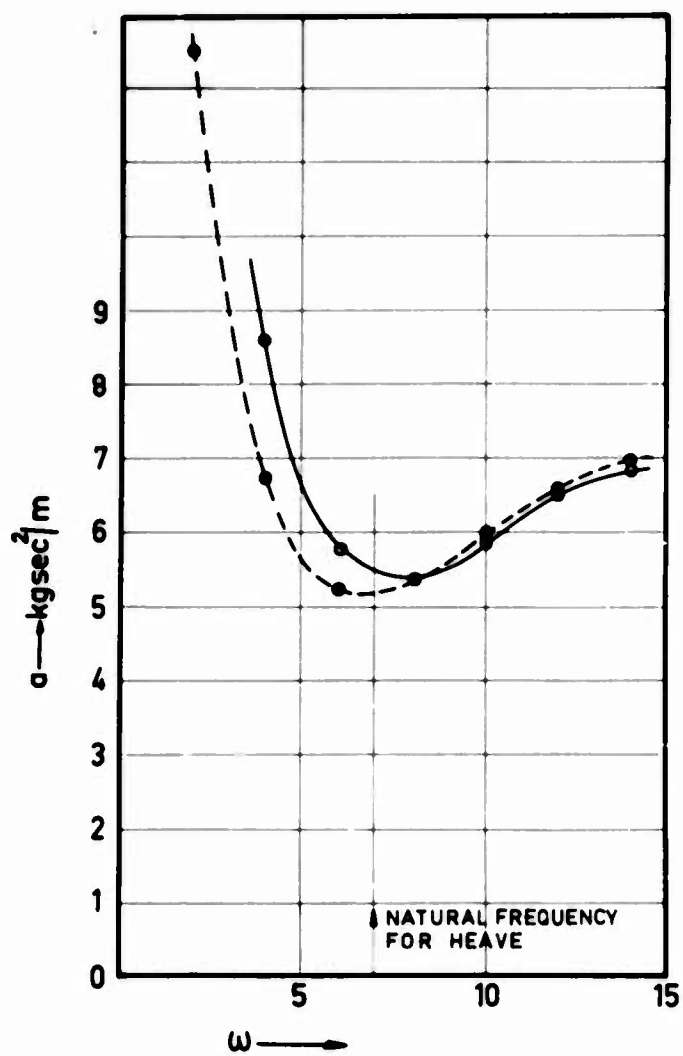


FIG 7 DISTRIBUTION OF a, b, d AND e OVER THE LENGTH OF THE SHIPMODEL



$F_n = 0$
 —●— EXPERIMENT
 -◆- CALCULATION

FIG. 8 COMPARISON OF THE CALCULATED VALUES OF a, b, D AND E WITH THE EXPERIMENTS

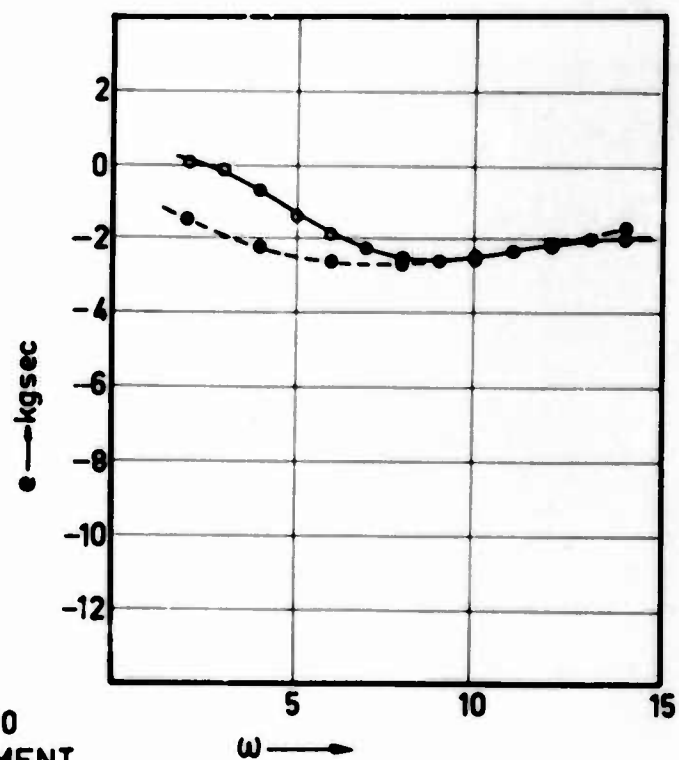
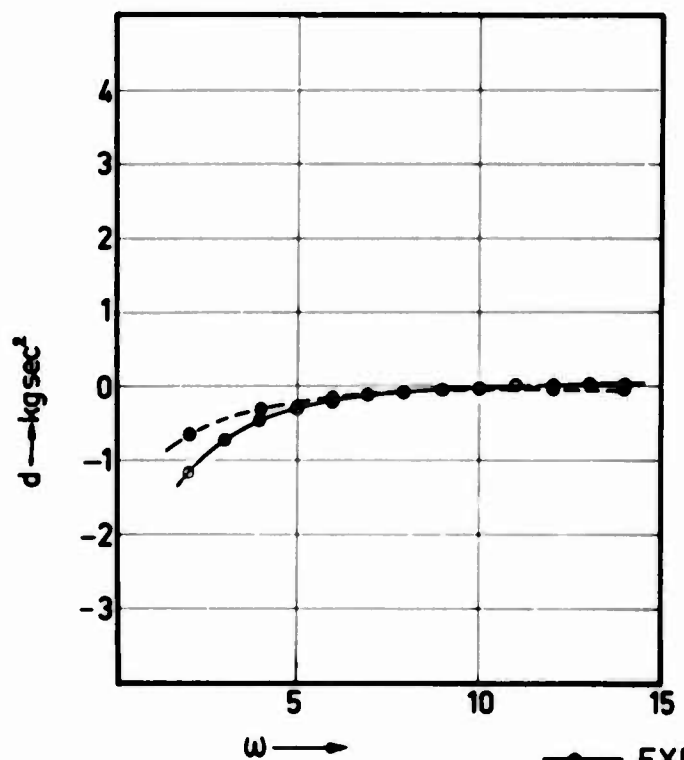
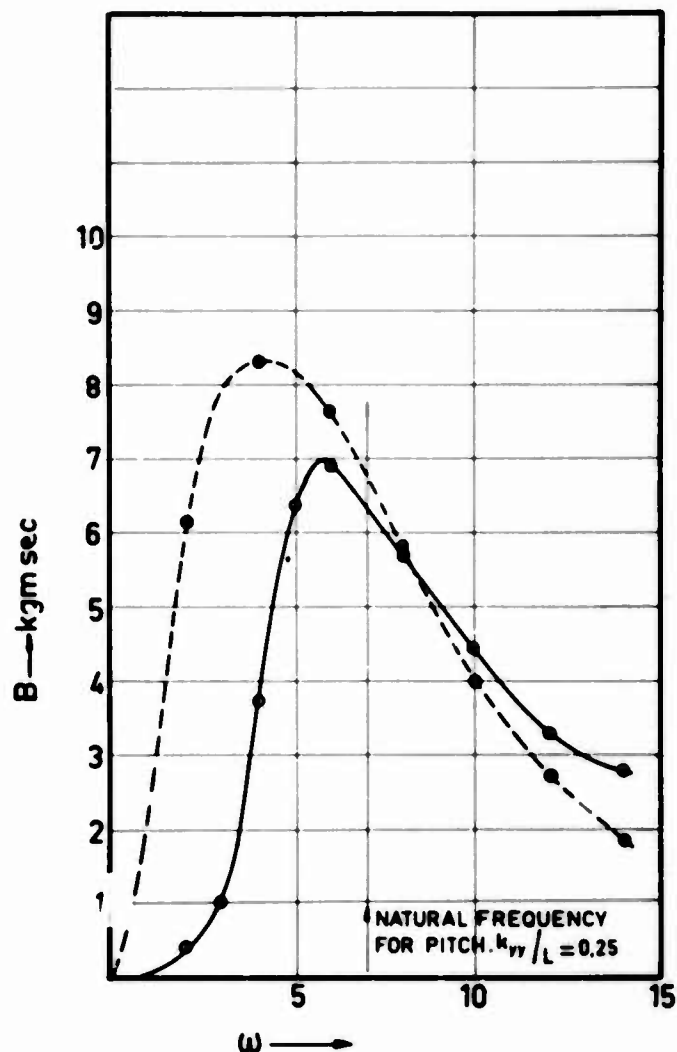
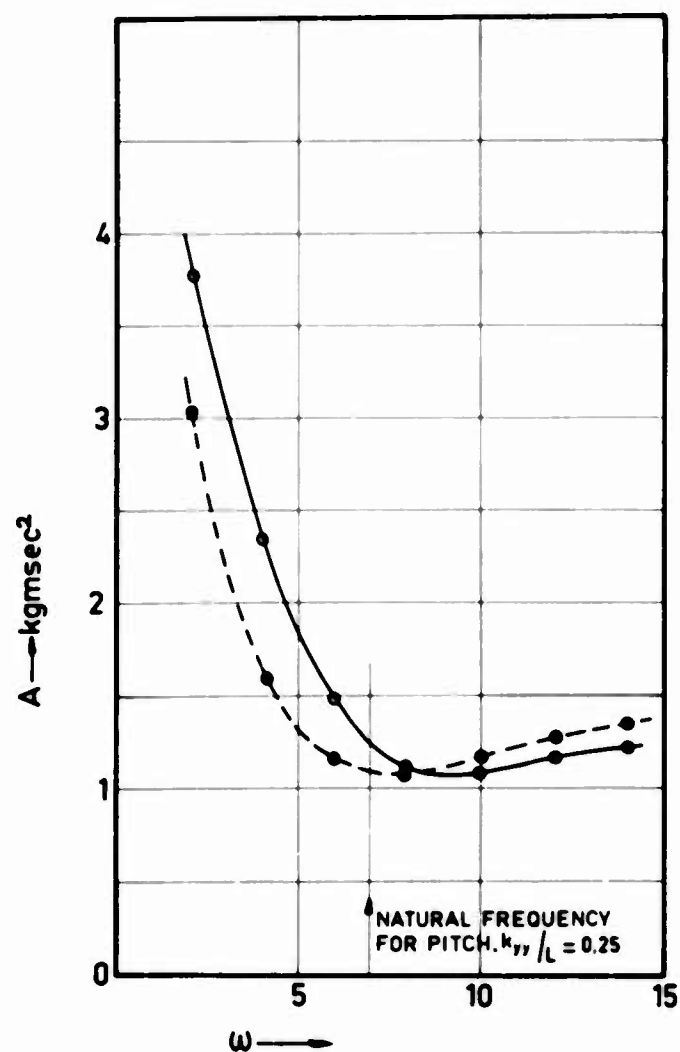


FIG.9 COMPARISON OF THE CALCULATED VALUES OF A,B,d AND e WITH THE EXPERIMENTS

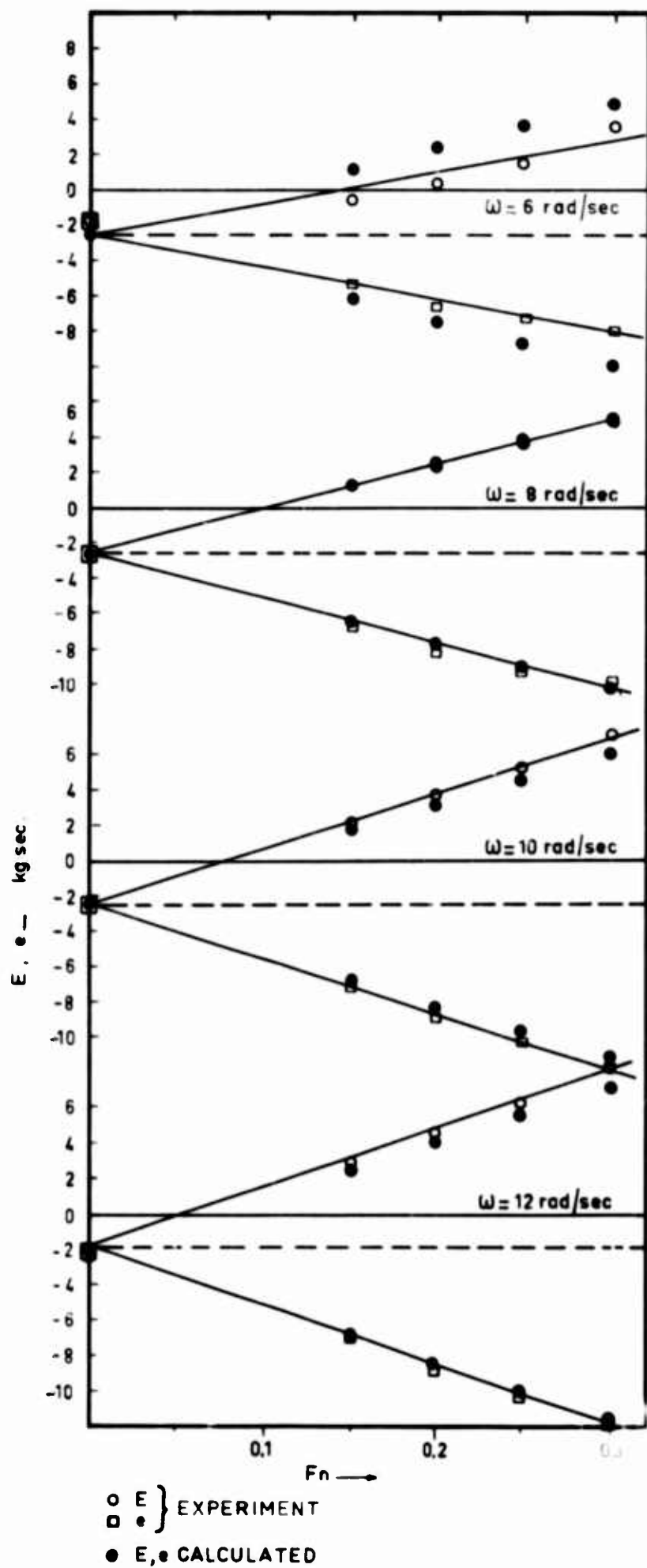


FIG 10 DAMPING CROSS COUPLING COEFFICIENT AS A FUNCTION OF FORWARD SPEED